MATH 2250 Calculus I
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Name: Answers
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Practice Quiz No. 6

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 Given the function $f(x) = 2x^3 + 3x^2 + 5$

a *Find all critical points of $f(x)$.*

$f'(x) = 6x^2 + 6x$, polynomial $\Rightarrow$ always exists

$\Rightarrow$ Set $f'(x) = 0 \Rightarrow 0 = 6x(x+1) \Rightarrow x = 0, -1$

are the only critical points.

b *Find the minimum and maximum of $f(x)$ on the interval $[-2, 1]$*

Check critical points and endpoints:

x	$f(x)$
-2	$2(-2)^3 + 3(-2)^2 + 5 = -16 + 12 + 5 = 1 = \min_{x \in [-2,1]} (f(x))$
-1	$2(-1)^3 + 3(-1)^2 + 5 = -2 + 3 + 5 = 6$
0	$2(0)^3 + 3(0)^2 + 5 = 5$
1	$2(1)^3 + 3(1)^2 + 5 = 10 = \max_{x \in [-2,1]} (f(x))$

Problem 2 Given the function $f(x) = x^2|x-2|$

a *Find all critical points of $f(x)$.*

When $x > 2$, $f(x) = x^2(x-2) \Rightarrow f'(x) = 3x^2 - 4x$
$= x(3x-4)$

$\Rightarrow x = 0, \frac{4}{3}$ would be critical points, but neither is > 2

When $x < 2$, $f(x) = x^2(-(x-2)) = -x^3 + 2x^2 \Rightarrow f'(x) = -3x^2 + 4x$

$\Rightarrow f'(x) = x(-3x+4) \Rightarrow x = 0, \frac{4}{3}$, are critical points because both are > 2.

When $x = 2$, the left are right derivatives are not equal, so the derivative does not exist. So $x = 2$ is a critical po[int]

b *Find the minimum and maximum of $f(x)$ on the interval* $[1, 3]$

Check endpoints and critical points:

x	$f(x)$
1	$(1)^2\|1-2\| = 1$
$\frac{4}{3}$	$\left(\frac{4}{3}\right)^2 \left\|\frac{4}{3} - 2\right\| = \left(\frac{16}{9}\right)\left\|\frac{4}{3} - \frac{6}{3}\right\| = \frac{16}{3}\left(\frac{2}{3}\right) = \frac{32}{9} < 9$
2	$(2)^2\|2-2\| = 0 = \min_{x\in[1,3]}(f(x))$
3	$(3)^2\|3-2\| = 9 = \max_{x\in[1,3]}(f(x))$

(0 is not in $[1,3]$)

Problem 3 Given the function $f(x) = \sqrt{1+x^2}$

a *Find all critical points of $f(x)$.*

$$f'(x) = \frac{1}{2}(1+x^2)^{-1/2}(2x) = \frac{x}{\sqrt{1+x^2}}$$

$1+x^2 \geq 1 > 0$, so $f'(x)$ always exists

$\Rightarrow$ Set $f'(x) = 0 \Rightarrow \frac{x}{\sqrt{1+x^2}} = 0 \Rightarrow x = 0$

is the only critical point.

b *Find the minimum and maximum of $f(x)$ on the interval $[-1, 1]$*

Check endpoints and critical points:

x	$f(x)$	
-1	$\sqrt{1+(-1)^2} = \sqrt{2}$	$= \max\limits_{x\in[-1,1]}(f(x))$
0	$\sqrt{1+(0)^2} = 1$	$= \min\limits_{x\in[-1,1]}(f(x))$
1	$\sqrt{1+(1)^2} = \sqrt{2}$	

This function has two values of $\underset{x\in[-1,1]}{\arg\max}(f(x))$

Problem 4 Given the function $f(x) = x^{2/3} - 2x^2$

a *Find all critical points of $f(x)$.*

$$f'(x) = \frac{2}{3}x^{-1/3} - 4x = \frac{2}{3x^{1/3}} - 4x$$

$f'(x)$ does not exist when $x = 0$, so $x=0$ is a critical poi

Set $f'(x) = 0 \Rightarrow \frac{2}{3x^{1/3}} - 4x = 0$, assuming $x \neq 0$.

$$\Rightarrow \frac{2}{3x^{1/3}} = 4x \Rightarrow \frac{2}{12} = x^{4/3} \Rightarrow x = \pm\left(\frac{1}{6}\right)^{3}$$

So the critical points are $x = 0, \pm\left(\frac{1}{6}\right)^{3/4}$

b *Find the minimum and maximum of $f(x)$ on the interval $[-1, 1]$*

Check endpoints and critical points:

x	$f(x)$
$-\left(\frac{1}{6}\right)^{3/4}$	$\approx .260847 = \max\limits_{x \in [-1,1]} (f(x))$
0	$0 = \min\limits_{x \in [-1,1]} (f(x))$
$\left(\frac{1}{6}\right)^{3/4}$	$\approx .260847$